A Z-Transform-Based Absorbing Boundary Conditions for 3-D TLM-SCN Method

Zhenhai Shao, Wei Hong, and Hongwei Wu

***Abstract*—In this paper, an efficient absorbing boundary condition (ABC) constructed in the Z-transform domain [i.e., Z-transform-based absorbing boundary conditions (Z-ABCs)] for the three-dimensional symmetrical condensed-node transmission-line matrix method is presented. Numerical results indicate that the Z-ABCs show better performance than the conventional Higdon's ABC in suppressing instability caused by spurious modes.**

***Index Terms*—3-D SCN-TLM, Z-ABCs.**

I. Introduction

The three-dimensional symmetrical condensed-node transmission-line matrix (3-D SCN-TLM) method [1] has been extensively applied to the analysis of electromagnetic problems with complex geometry such as microwave circuits, etc. For open-region problems, absorbing boundary conditions (ABCs) are very important in obtaining a virtual extension of the limited computational domain [1]. Generally, there are two ways to construct ABCs. One is the employment of nonphysical absorbing media, such as the perfect matched layer (PML) [2], which can absorb the wide-angle scattered wave perfectly, but is complicated in formulation and normally needs extra computer memories. Another is the usage of outgoing wave equations [3]–[8], which only use the fields at the neighboring space and time nodes and does not require extra computer memories.

In this paper, a new kind of ABC derived from the Z-transform domain [7] is presented and applied to the 3-D TLM-SCN, which shows better performance and stability in damping spurious reflection compared with the conventional standard Higdon's ABC (S-Higdon's ABC) [3], [4], [8] or the modified Higdon's ABC (M-Higdon's ABC).

II. Formulation of the ABCs

A. Basic Theory

A scattered wave propagating along the $-x$-direction with the phase velocity v is assumed and the truncated boundary is set at $x = 0$. In the Z-transform domain, the relation between the field $E_0(z)$ at the boundary node $x_0 = 0$ and the fields $E_k(z)$, $k = 1, 2, \ldots, p$ at the interior nodes $x_k = k\triangle x$ can be expressed as [7]

$$E_0(z) = \sum_{k=1}^{p} h_k(z) E_k(z) \tag{1}$$

where p is the order of the ABC and $h_k(z)$, $k = 1, 2, \ldots, p$ are the transfer functions that satisfy

$$h_k = (-1)^{k+1} \sum_{\substack{1 \le k_1, \ldots, k_k \le p \\ k_i \ne k_j, 1 \le i, j \le k}} (d_{k_1} d_{k_2}, \ldots, d_{k_k}), \qquad k = 1, 2, \ldots, p \tag{2}$$

with $d = z^{-s}$, $s = \triangle x/(v\triangle t)$, $\triangle x$, and $\triangle t$ being the space and time increments, respectively.

In practice, the phase velocity v is determined by the incident angle θ (the angle between the incident direction and the x-axis) of the scattered wave or the effective dielectric constant ε_{eff} of the transmission lines, i.e., $v = v_0/\cos\theta$ or $v = v_0/\sqrt{\varepsilon_{\text{eff}}}$, where v_0 denotes the light velocity of free space.

Normally, the factor s in the transfer functions is not an integer, which means that time delay of a space increment is not integer times of the time increment. Consequently, (1) and (2) cannot be changed into difference schemes directly. In practical applications, proper simplification is necessary. In [7], two kinds of ABCs are obtained by expanding z^{-s} to a rational polynomial of z^{-1} for the finite-difference time-domain (FDTD) method. In [3], [4], and [8], it is pointed out that the transmission-line matrix (TLM) method cannot obtain stable results with an S-Higdon's ABC because of spurious modes. In the TLM method, the scattered impulse on the link line of a node resulted from an incident impulse with the same polarization on the link line at the other side of the node only appears after two time steps $2\triangle t$. Thus, we propose to expand z^{-s} to a rational polynomial of z^{-2}. This ABC does not produce spurious modes and shows better performance than the Higdon's ABC. The details are shown as follows:

$$\begin{aligned} z^{-s} &= \frac{z^{-\alpha-\beta s}}{z^{-\alpha+(1-\beta)s}} = \frac{(z^{-2})^{(\alpha+\beta s)/2}}{(z^{-2})^{[\alpha-(1-\beta)s]/2}} \\ &\approx \frac{1 + (\alpha+\beta s)\dfrac{(z^{-2}-1)}{2}}{1 + \Big[\alpha - (1-\beta)s\Big]\dfrac{(z^{-2}-1)}{2}} \\ &= \frac{\Big[2 - (\alpha+\beta s)\Big] + (\alpha+\beta s) z^{-2}}{\Big\{2 - \Big[\alpha - (1-\beta)s\Big]\Big\} + \Big[\alpha - (1-\beta)s\Big] z^{-2}}. \end{aligned} \tag{3}$$

Applying (3) and (2) to (1) and using the inversion of the Z-transform, the Z-transform-based absorbing boundary conditions (Z-ABCs) for the TLM method can be obtained in time domain.

In order to use the pth order Z-ABCs constructed from (3), its coefficients must be given, which can be expressed as

$$\sum_{i=0}^{p} C_{0,i}^{p} E_0^{n-2i} = \sum_{i=1}^{p} \sum_{j=0}^{p} C_{i,j}^{p} E_i^{n-2j}. \tag{4}$$

Recursive formulas for the coefficients in (4) can be derived as

$$C_{0,0}^{1} = b_1 \tag{5}$$
$$C_{0,1}^{1} = a_1 \tag{6}$$
$$C_{1,0}^{1} = d_1 \tag{7}$$
$$C_{1,1}^{1} = c_1 \tag{8}$$
$$C_{0,0}^{p} = C_{0,0}^{p-1} b_p \tag{9}$$
$$C_{0,i}^{p} = C_{0,i-1}^{p-1} + C_{0,i}^{p-1} b_p, \qquad i = 1, 2, \ldots, p-1 \tag{10}$$
$$C_{0,p}^{p} = C_{0,p-1}^{p-1} a_p \tag{11}$$
$$C_{1,0}^{p} = C_{0,0}^{p-1} d_p + C_{1,0}^{p-1} b_p \tag{12}$$
$$C_{1,j}^{p} = C_{0,j-1}^{p-1} c_p + C_{0,j}^{p-1} d_p + C_{1,j-1}^{p-1} a_p + C_{1,j}^{p-1} b_p, \quad 1 \le j < p \tag{13}$$
$$C_{1,p}^{p} = C_{0,p-1}^{p-1} c_p + C_{1,p-1}^{p-1} a_p \tag{14}$$
$$C_{k,0}^{p} = C_{k,0}^{p-1} b_p - C_{k-1,0}^{p-1} d_p, \qquad k = 2, 3, \ldots, p-1 \tag{15}$$
$$C_{k,j}^{p} = C_{k,j-1}^{p-1} a_p + C_{k,j}^{p-1} b_p - C_{k-1,j-1}^{p-1} c_p - C_{k-1,j}^{p-1} d_p, \quad 2 \le k < p; \quad 1 < j < p \tag{16}$$
$$C_{k,p}^{p} = C_{k,p-1}^{p-1} a_p - C_{k-1,p-1}^{p-1} c_p, \qquad 2 \le k < p \tag{17}$$
$$C_{p,0}^{p} = -C_{p-1,0}^{p-1} d_p \tag{18}$$

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$$C_{p,i}^{p} = -C_{p-1,i-1}^{p-1}c_p - C_{p-1,i}^{p-1}d_p, \qquad 1 \le i < p \tag{19}$$
$$C_{p,p}^{p} = -C_{p-1,p-1}^{p-1}c_p \tag{20}$$

where

$$a_i = \alpha - (1-\beta)s_i, \quad i = 1, 2, \ldots, p \tag{21}$$
$$b_i = 2 - \Big[\alpha - (1-\beta)s_i\Big], \qquad i = 1, 2 \ldots, p \tag{22}$$
$$c_i = \alpha + \beta s_i, \qquad i = 1, 2, \ldots, p \tag{23}$$
$$d_i = 2 - (\alpha + \beta s_i), \qquad i = 1, 2, \ldots, p. \tag{24}$$

B. Stability Analysis

The stability condition for the Z-ABCs can be obtained by the Von Neumann (Fourier series) method. The Z-ABCs of the first order can be written as

$$\Big[2 + s - (\alpha + \beta s)\Big]E_0^n + \Big[(\alpha + \beta s) - s\Big]E_0^{n-2} = \Big[2 - (\alpha + \beta s)\Big]E_1^n + (\alpha + \beta s)E_1^{n-2}. \tag{25}$$

Using the Von Neumann method, the unconditional stability condition for (25) can be derived as

$$\alpha + \beta s < 1 + \frac{s}{2}. \tag{26}$$

The conditional stability condition for (25) can be obtained as

$$\alpha + \beta s = 1 + \frac{s}{2}. \tag{27}$$

For $\alpha = \beta = 0.5$, the Z-ABCs are unconditional stable. This condition is also suitable for the Z-ABCs of the second or higher order.

III. Numerical Results

In order to assess the effectiveness of the Z-ABCs, some problems of rectangular-waveguide, microstrip transmission-line, low-pass, and bandpass filters are analyzed by using the 3-D TLM-SCN method with the proposed Z-ABCs. In the following examples, we always choose the following scheme in truncated boundaries: the tangential electric fields and normal magnetic fields in truncated boundary are all absorbed by the Z-ABCs. When using this scheme, the corner and edge of the truncated boundary will need not be considered.

First, let us consider a WR28 rectangular waveguide in which the waves can be considered as a superposition of many plane waves bouncing back and forth on the wall at different incident angles. Therefore, behavior of the wide-angle absorption of the ABCs can be evaluated. Simulations were performed using the 3-D TLM-SCN method. Both ends of the waveguide were terminated with the Z-ABCs and the conventional Higdon's ABCs. The voltage standing-wave ratio (VSWR) in the waveguide is computed directly and numerically with the ratio of $V_{\max}$ over $V_{\min}$, where V is the amplitude of the dominant mode in the waveguide. Fig. 1 shows the return loss computed by the 3-D TLM-SCN method with Z-ABCs and Higdon's ABCs, respectively. It can be observed that the Z-ABCs shows better performance than the S-Higdon's ABC and M-Higdon's ABC in most frequency ranges.

Secondly, consider a microstrip, the thickness and dielectric constant of its substrate are 0.1 mm and 13, respectively. The width of the metal strip is 0.15 mm (due to the symmetry of the problem, only one-half of the structure is considered). The microstrip is excited by a Gaussian pulse at the center point of the cross section under the strip and only E_y component is assumed as

$$E_y^i = \exp\left[-\frac{(t-t_0)^2}{T^2}\right] \tag{28}$$

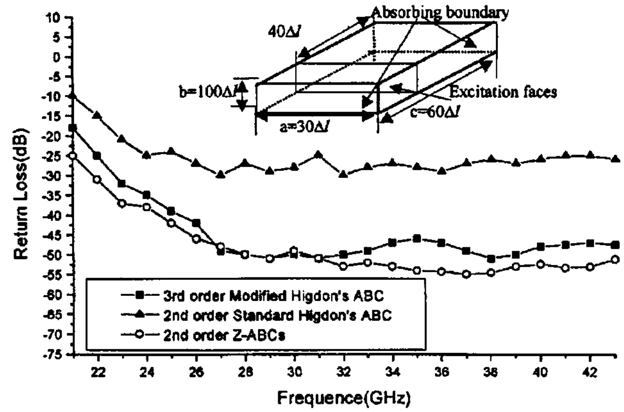


Fig. 1. Comparison of the return losses in a rectangular waveguide.

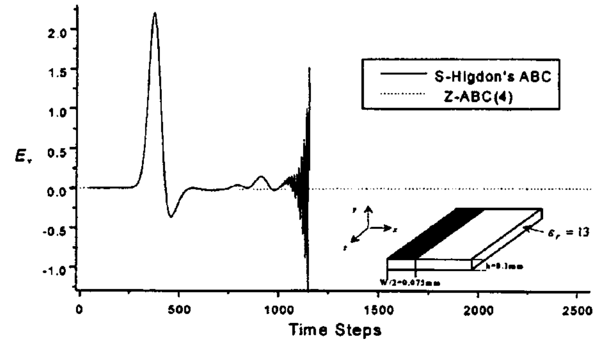


Fig. 2. Comparison of the electric field at the center point under the strip by the TLM method with the Z-ABC and Jigdon's ABC, where $\alpha = \beta = 0.5$.

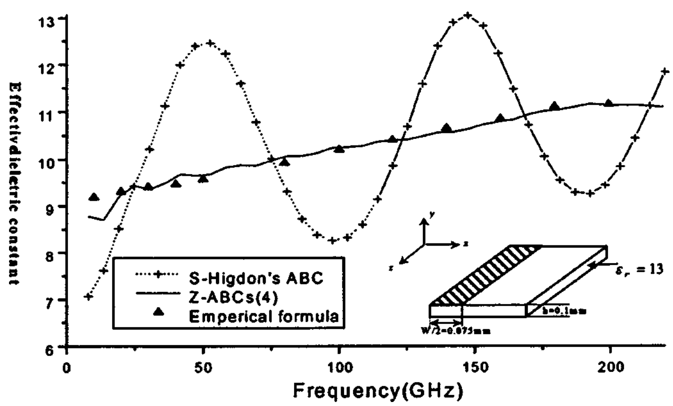


Fig. 3. Comparison of the effective dielectric constant of a microstrip with Z-ABCs and the Higdon's ABCs and empirical formula.

where $t_0 = 164\triangle t$ and $T = 42\triangle t$. There are 119 mesh nodes along the z-direction, 35 nodes away from the metal strip in the x-direction and 11 nodes in the air layer along the y-direction. Along the z-direction, the Z-ABCs of the second order are used, in which $v_1 = v_0/\sqrt{9}$ and $v_1 = v_0/\sqrt{11}$. Along the x- and y-directions, the Z-ABCs of the first order are used, in which $v = v_0/(\sqrt{9.12}\cos\theta)$, where $\theta = 30^\circ$ or 0° corresponding to the x- or y-directions, respectively. The same parameters for the S-Higdon's condition are chosen.

When $\alpha = \beta = 0.5$, Fig. 2 shows the comparison of electric field at the center point under the strip. For S-Higdon's ABCs, E_y is divergent after 1200 time steps. However, E_y is still convergent after 7000 time steps with the Z-ABCs.

When $\alpha = \beta = 0.25$, Figs. 3 and 4 show the comparison of the effective dielectric constant and electric field E_y at the center point

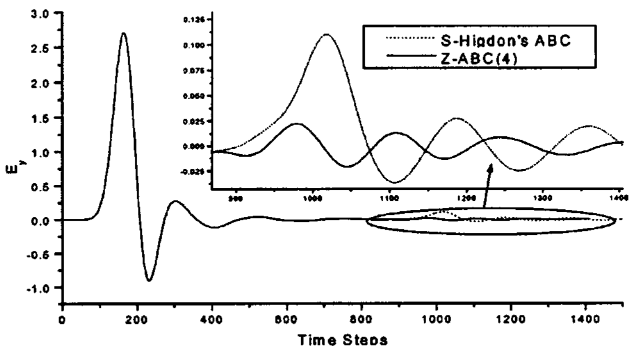


Fig. 4. Comparison of the electric field at the center point under the strip with the Higdon's ABC and Z-ABCs.

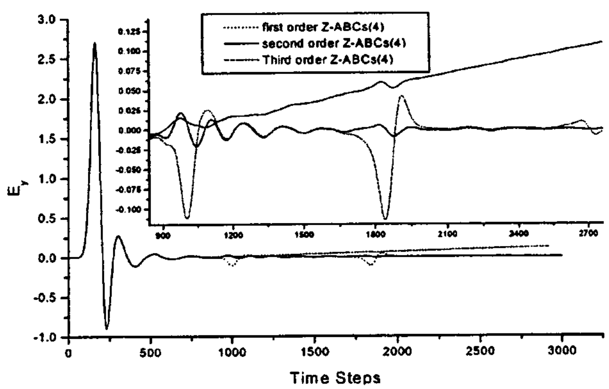


Fig. 5. Comparison of the electric field at the center of the reference section under the strip with the Z-ABCs of the first, second, and third orders in the z-direction, respectively.

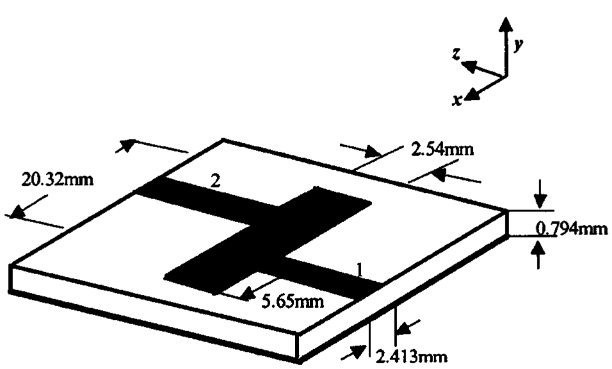
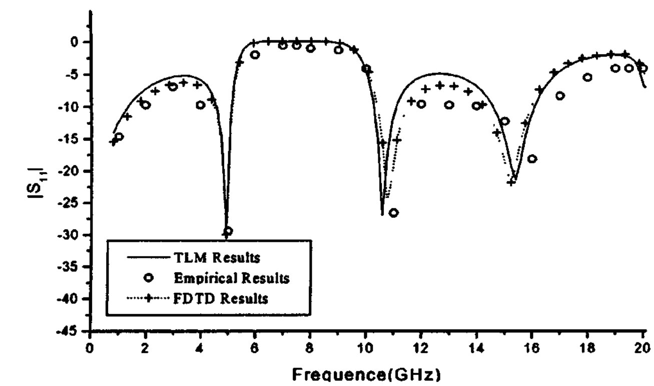


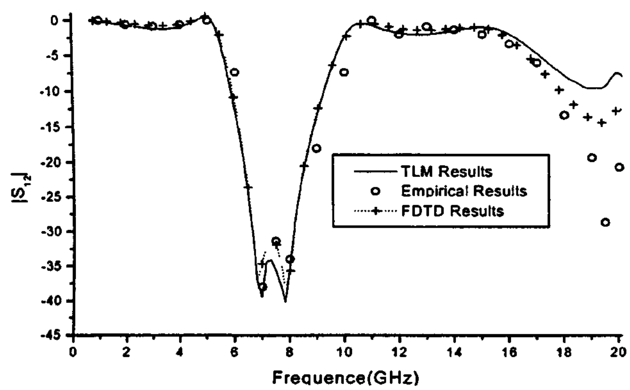
Fig. 6. Configuration of a microstrip low-pass filter.

under the strip with the Z-ABCs and S-Higdon's ABCs, respectively. It can be seen that the results with the Z-ABCs are in better agreement with empirical results than those with S-Higdon's ABCs. It can be seen that the vertical electric field reflected from boundaries with the Z-ABCs is reduced more than 200%–300% compared to the results with S-Higdon's ABCs. Fig. 5 shows the comparison of the electric field E_y at the center of the reference plane under the strip with the Z-ABCs of the first, second, and third orders in the z-direction, respectively. It is obvious that the effectiveness with the Z-ABCs of the second order is better than with the Z-ABCs of the first order. Due to numerical dispersion, the effectiveness with Z-ABCs of the third order is worse than Z-ABCs of the second order.

Next, consider a microstrip low-pass filter printed on a Duriod ($\varepsilon_r = 2.2$) dielectric substrate, as shown in Fig. 6. The mesh spaces used here are $\triangle x = 0.4064$ mm, $\triangle y = 0.265$ mm, and



(a)



(b)

Fig. 7. Scattering parameters of the low-pass filter. (a) Return loss. (b) Insertion loss.

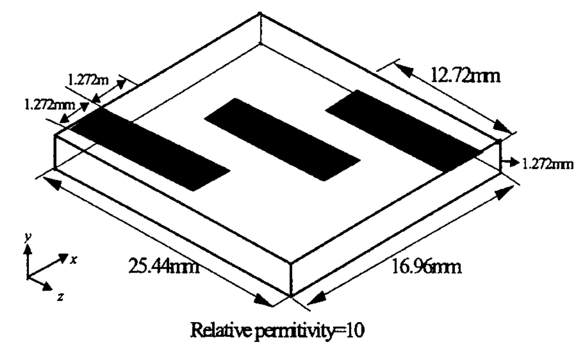


Fig. 8. Configuration of a microstrip bandpass filter.

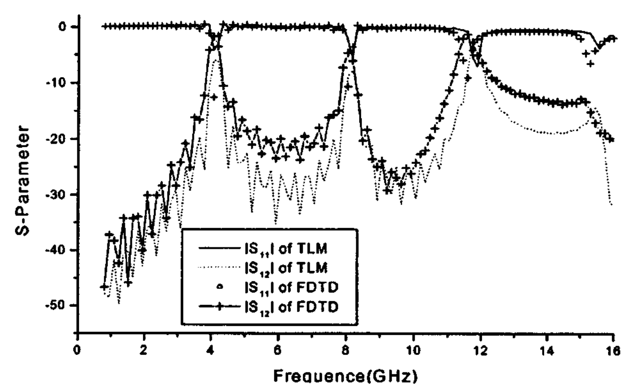


Fig. 9. Scattering parameters of the bandpass filter.

$\triangle z = 0.4233$ mm. Along the z-direction, the Z-ABCs of the second order are used, in which $v_1 = v_0/\sqrt{1.7}$ and $v_2 = v_0/\sqrt{2.2}$. Along the

x- and y-directions, the Z-ABCs of the first order are used, in which $v = v_0/(\sqrt{1.89}\cos\theta)$ and θ is 30° or 0° corresponding to the x- and y-directions, respectively. The scattering parameters computed with both the TLM and FDTD methods are shown in Fig. 7.

Finally, consider a microstrip bandpass filter (Fig. 8). The computed scattering parameters are shown in Fig. 9 by using the TLM-SCN method with Z-ABCs. Along the z-direction, Z-ABCs of the second order are used, in which $v_1 = v_0/\sqrt{6.6}$ and $v_2 = v_0/\sqrt{9.5}$. Along the x- and y-directions, Z-ABCs of the first order are used, in which $v = v_0/(\sqrt{6.85}\cos\theta)$ and θ is 30° or 0° corresponding to the x- and y-directions, respectively. The results are in good agreement with the results of the FDTD method, where the uniform mesh $\triangle x = \triangle y = \triangle z = 0.318$ mm is used.

IV. Conclusion

In this paper, an efficient Z-ABC constructed in the Z-transform domain has been proposed for the 3-D TLM-SCN. Numerical results for the transmission problems of the waveguide and microstrip, as well as the discontinuity problems of microstrip low-pass and bandpass filters, verified the validity of Z-ABCs.

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